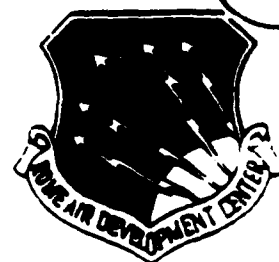


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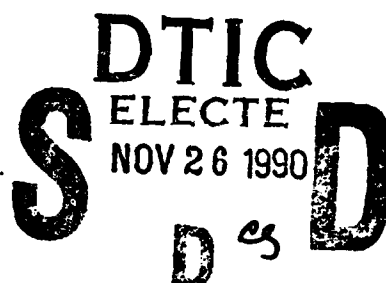


AD-A228 811

RADC-TR-90-249
Final Technical Report
September 1990

A GENERALIZED TREATMENT OF MUTUAL COUPLING COMPENSATION

Syracuse University



Braham Himed and Donald D. Weiner

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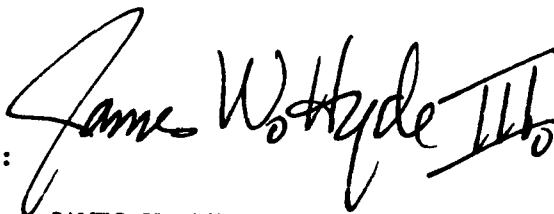
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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE October 1990		3. REPORT TYPE AND DATES COVERED Final Jan 88 - Jan 89	
4. TITLE AND SUBTITLE A GENERALIZED TREATMENT OF MUTUAL COUPLING COMPENSATION				5. FUNDING NUMBERS C - F30602-88-D-0027 PE - 62702F PR - 4506 TA - 11 WU - P1	
6. AUTHOR(S) Braham Himed, Donald D. Wiener					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Syracuse University Syracuse NY 13244				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Rome Air Development Center (OCTS) Griffiss AFB NY 13441-5700				10. SPONSORING/MONITORING AGENCY REPORT NUMBER RADC-TR-90-249	
11. SUPPLEMENTARY NOTES RADC Project Engineer: Vincent Vannicola/OCTS/(315) 330-4437					
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) The ESPRIT algorithm has been shown to be an effective solution to the angle of arrival estimation problem. One possibility for the implementation of ESPRIT is the use of a linear array to provide for the sensor paris. This paper discusses a technique for compensation of mutual coupling effects between array elements. Computer simulations demonstrate a significant improvement in performance.					
14. SUBJECT TERMS ESPRIT algorithm, linear array, wavelength dipoles				15. NUMBER OF PAGES 12	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL		

A Generalized Treatment of Mutual Coupling Compensation
for ESPRIT

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Accession For	
NTIS CRAM	✓
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Abstract

The ESPRIT algorithm has been shown to be an effective solution to the angle of arrival estimation problem [1]. One possibility for the implementation of ESPRIT is the use of a linear array to provide for the sensor pairs. This paper discusses a technique for compensation of mutual coupling effects between array elements. Computer simulations demonstrate a significant improvement in performance.

Introduction

Direction finding, which involves estimation of the angles of arrival of sources, is very important in many sensor systems such as radar, sonar, seismology, etc. Several authors have approached the problem using subspace methods [1,2]. However, these methods have not taken into account effects of mutual coupling between array elements which can significantly deteriorate the eigensystems underlying the solution procedures. In this paper we deal with compensation of the mutual coupling effects when a linear array consisting of m sensors is used in conjunction with the ESPRIT algorithm [1]. The method of moments [3,4] is used to obtain the matrix of mutuals for each sensor pair. A transformation matrix is developed which processes the observed data so as to estimate the signals that would have resulted had there been no mutuals. We show that ideally the effects of mutual coupling can be completely eliminated. Computer simulations demonstrating the improved performance are presented.

Mutual Coupling

Consider a linear array of m dipoles uniformly spaced at a distance D . Each dipole is of length l and has a radius r satisfying the condition $r \ll l$. A load is attached to the center gap of each dipole. Assume there are d narrowband signals impinging on the array as planar wavefronts. The voltages induced by the assumed signals on the loads are the outputs of the dipoles. Induced currents will appear on the dipoles. These currents reradiate and generate scattered fields. The scattered fields then induce currents on the neighboring dipoles. The process of induction and reradiation causes mutual coupling between the dipoles. Using single sinusoidal expansion and weighting functions per dipole, the method of moments [3,4]

is employed to obtain the matrix of mutuals. Denote the current distribution in the array of dipoles by $J(z)$ (assuming longitudinal distribution and neglecting all other distributions) and the j -th expansion function by $f_j(z)$. Then

$$J(z) = \sum_{j=1}^m I(j) f_j(z) \quad (1)$$

where $I(j)$ are unknown amplitudes to be determined. At a point (y, z) in the Y - Z plane, the scattered field is given by

$$E(s)(y, z) = \sum_{j=1}^m I(j) E_j^{(s)}(y, z) \quad (2)$$

where $E_j^{(s)}(y, z)$ is the scattered field from the j th dipole. The total field is

$$E(y, z) = E^{(inc)}(y, z) + E(s)(y, z) \quad (3)$$

where $E^{(inc)}$ is the incident field. Let E_z be the z -component of the total field. A generalized voltage $V(i)$ induced on the subsection spanned by the function $f_i(z)$ can be defined with respect to a weighting function $w_i(z)$ as

$$V(i) = F(E_z(y, z), w_i(z)) \quad (4)$$

where F is bilinear with respect to $E_z(y, z)$ and $w_i(y, z)$. Similarly, we define

$$V^{(inc)}(i) = F(E_z^{(inc)}(y, z), w_i(z)), \quad (5)$$

$$V^{(s)}(i) = F(E_z^{(s)}(y, z), w_i(z)). \quad (6)$$

Thus,

$$V(i) = V^{(inc)}(i) + V^{(s)}(i),$$

which, for metallic scatterers, becomes

$$V(i) = V^{(inc)}(i) + V^{(s)}(i) = 0,$$

$$V^{(inc)}(i) = -V^{(s)}(i). \quad (7)$$

However,

$$V^{(s)}(i) = F\left(\sum_{j=1}^m I(j) E_j^{(s)}(y, z), w_i(z)\right)$$

$$= \sum_{j=1}^m I(j) F(E^{(j)}(y, z), w_i(z)) .$$

Let

$$z^{ij} = -F(E^{(j)}(y, z), w_i(z)) . \quad (8)$$

Then

$$v^{(s)}(i) = \sum_{j=1}^m -z^{ij} I(j) ; i=1, 2, \dots, m. \quad (9)$$

In matrix notation $\underline{v}^{(s)} = -Z \underline{I}$ where

$$\underline{v}^{(s)T} = [v^{(s)}(1), v^{(s)}(2), \dots, v^{(s)}(m)]$$

and

$$\underline{I}^T = [I(1), I(2), \dots, I(m)] .$$

The matrix Z can be decomposed into two parts as $Z = Z_0 + Z_L$, where

Z_0 is the generalized impedance matrix

and

Z_L is the load matrix.

Assuming that all dipoles are loaded with the same load z_l , the matrix Z_L is given by

$$Z_L = \text{diag}\{z_l, z_l, \dots, z_l\} .$$

The ij -th element of Z is $z^{ij} = z_{ij} + z_l \delta_{ij}$. The voltages induced on a load z_l are given by

$$\underline{v}^{(t)} = Z_L \underline{I} \text{ and } \underline{I} = Z_L^{-1} \underline{v}^{(t)} .$$

However,

$$\underline{v}^{(inc)} = Z \underline{I} = Z_0 Z_L^{-1} \underline{v}^{(t)} + \underline{v}^{(t)} ,$$

which implies that

$$\underline{v}^{(t)} = [I + Z_0 Z_L^{-1}]^{-1} \underline{v}^{(inc)} . \quad (10)$$

Let H be the matrix

$$H = [I + Z_0 Z_L^{-1}] . \quad (11)$$

Thus, when incident signals are impinging on the array and in the presence of additive noise, the output of the linear array will be

$$\underline{v}^{(t)} = H^{-1} \underline{v}^{(inc)} + \underline{N} .$$

For simplicity, let $\underline{X} = \underline{v}^{(inc)}$ and $\underline{Y} = \underline{v}^{(t)}$. We now have a relationship between the incident signals and the received signals at the outputs of the array, which is

$$\underline{Y} = H^{-1} \underline{X} + \underline{N} . \quad (12)$$

APPLICATION TO ESPRIT

Consider a linear array of $(m+1)$ sensors and assume there are d ($d < m$) narrowband sources located at angles θ_k ; $k=1, \dots, d$.

First Application

In the first application we consider two sub-arrays consisting of the first m sensors and the last m sensors. The observed signal vector at the output of the array can be written as

$$\underline{Y} = H^{-1} \underline{X} + \underline{N} . \quad (13)$$

Let $G = H^{-1}$. G can be written as

$$\begin{bmatrix} g_{11} & g_{12} & \dots & g_{1(m+1)} \\ g_{21} & g_{22} & \dots & g_{2(m+1)} \\ \vdots & \vdots & \ddots & \vdots \\ g_{(m+1)1} & g_{(m+1)2} & \dots & g_{(m+1)(m+1)} \end{bmatrix}$$

Thus if

$$\underline{Y}_1 = [y_1, y_2, \dots, y_m]^T ,$$

and

$$\underline{Y}_2 = [y_2, y_3, \dots, y_{(m+1)}]^T ,$$

we can write

$$\underline{Y}_1 = G_{11} \underline{X}_1 + G_{12} \underline{X}_2 + \underline{N}_1 \quad (14)$$

$$\text{and } \underline{Y}_2 = G_{21} \underline{X}_1 + G_{22} \underline{X}_2 + \underline{N}_2 , \quad (15)$$

where G_{11} , G_{12} , G_{21} , G_{22} , \underline{N}_1 , \underline{N}_2 , \underline{X}_1 and \underline{X}_2 are given by

$$\underline{X}_1 = [x_1, x_2, \dots, x_m]^T ,$$

$$\underline{X}_2 = [x_2, x_3, \dots, x_{(m+1)}]^T .$$

$$G_{11} = [g_{111}, g_{112}, \dots, g_{11m}] ,$$

$$g_{11i} = [g_{11} \ g_{21} \ \dots \ g_{m1}]^T ; i=1, \dots, m ,$$

$$G_{12}^T = [0 \ 0 \ \dots \ 0 \ g_{11(m+1)}] ,$$

$$G_{21}^T = [g_{221} \ 0 \ \dots \ 0] ,$$

$$G_{22}^T = [g_{222} \ g_{223} \ \dots \ g_{22(m+1)}] ,$$

$$g_{22i} = [g_{21} \ g_{31} \ \dots \ g_{(m+1)1}]^T ; i=2, \dots, (m+1) ,$$

$$\underline{N}_1 = [n_1, n_2, \dots, n_m]^T ,$$

$$\underline{N}_2 = [n_2, n_3, \dots, n_{(m+1)}]^T .$$

Consider the vector \underline{Z} defined as

$$\underline{Z} = [\underline{Y}_1 \ \underline{Y}_2]^T .$$

\underline{Z} can be written as

$$\underline{Z} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \end{bmatrix} + \begin{bmatrix} \underline{N}_1 \\ \underline{N}_2 \end{bmatrix} . \quad (16)$$

Assuming that the signals and noise are statistically independent and that the noise components are uncorrelated from sensor to sensor with variance σ^2 , Then $C_{zz} = E[\underline{Z} \underline{Z}^H]$ is given by

$$C_{zz} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} E[\underline{X}_1 \underline{X}_1^H] & E[\underline{X}_1 \underline{X}_2^H] \\ E[\underline{X}_2 \underline{X}_1^H] & E[\underline{X}_2 \underline{X}_2^H] \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^H + \sigma^2 \begin{bmatrix} I_m & 0 \\ 0 & I_m \end{bmatrix} \quad (17)$$

Let $[G]$ and $[I]$ be the matrices

$$[G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad \text{and} \quad [I] = \begin{bmatrix} I_m & I_{1m} \\ I_{2m} & I_m \end{bmatrix}.$$

where I_m is the identity matrix and I_{1m} and I_{2m} are

$$I_{1m} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}$$

$$I_{2m} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}.$$

Then

$$[G]^{-1} (C_{zz} - \sigma^2 [I]) ([G]^{-1})^H = \begin{bmatrix} E[X_1 X_1^H] & E[X_1 X_2^H] \\ E[X_2 X_1^H] & E[X_2 X_2^H] \end{bmatrix}. \quad (18)$$

Having recovered the matrix on the right side of equation (18), the matrices $M = E[X_1 X_1^H]$ and $N = E[X_1 X_2^H]$ can be identified. It can be shown that M and N have the decompositions

$$M = ASA^H \quad \text{and} \quad N = AS\Phi A^H \quad (19)$$

where A , S and Φ are the following matrices

$$S = E[S S^H],$$

$$S^T = \{s_1, \dots, s_d\} \text{ impinging signal vector,}$$

$$A = [a_1 \ a_2 \ \dots \ a_d]$$

$$a_i = [1 \ e^{j\phi_i} \ \dots \ e^{j\phi_d}]^T,$$

$$\Phi = \text{diag} [e^{j\phi_1}, \dots, e^{j\phi_d}],$$

$$\phi_k = (\omega \Delta / c) \sin(\theta_k), \quad k=1, 2, \dots, d.$$

Therefore, the effects of mutual coupling have been eliminated and the rank reducing values of the matrix pencil $(M - \lambda N)$ are given by

$$\lambda_i = e^{j(\omega \Delta / c) \sin(\theta_i)}; \quad i=1, 2, \dots, d. \quad (20)$$

Second Application

In the second application, two neighboring sensors are considered as a doublet. Assume then we have a linear array of $2m$ sensors so as to constitute m doublets and let there be d ($d < m$) sources. Again, the received signal at the output of the array is modeled as

$$Y = G X + N \quad (21)$$

where G is given by

$$\begin{bmatrix} g_{11} & g_{12} & \dots & g_{1(2m)} \\ g_{21} & g_{22} & \dots & g_{2(2m)} \\ \vdots & \vdots & \ddots & \vdots \\ g_{(2m)1} & g_{(2m)2} & \dots & g_{(2m)(2m)} \end{bmatrix}$$

Let v_i and w_i be the signals received at the i -th doublet. Then

$$v_i = y_{(2i-1)} \quad \text{and} \quad w_i = y_{(2i)}. \quad (22)$$

Collecting all the v_i 's in a vector \underline{V} and all the w_i 's in a vector \underline{W} , we have

$$\underline{V} = G_{11} \underline{X}_1 + G_{12} \underline{X}_2 + \underline{N}_1 \quad (23)$$

and

$$\underline{W} = G_{21} \underline{X}_1 + G_{22} \underline{X}_2 + \underline{N}_2. \quad (24)$$

where G_{11} , G_{12} , G_{21} , G_{22} , \underline{X}_1 , \underline{X}_2 , \underline{N}_1 and \underline{N}_2 are given by

$$\underline{X}_1 = [x_1 \ x_3 \ \dots \ x_{(2m-1)}]^T,$$

$$\underline{X}_2 = [x_2 \ x_4 \ \dots \ x_{(2m)}]^T,$$

$$G_{11}^T = [g_{111} \ g_{112} \ \dots \ g_{11m}],$$

$$g_{11i} = \{g_{(2i-1)1} \ g_{(2i-1)3} \ \dots \ g_{(2i-1)(2m-1)}\};$$

$$i=1, 3, \dots, (2m-1),$$

$$G_{12}^T = [g_{121} \ g_{122} \ \dots \ g_{12m}],$$

$$g_{12i} = \{g_{(2i-1)2} \ g_{(2i-1)4} \ \dots \ g_{(2i-1)(2m)}\};$$

$$i=1, 3, \dots, (2m-1),$$

$$G_{21}^T = [g_{211} \ g_{212} \ \dots \ g_{21m}],$$

$$g_{21i} = \{g_{(2i)1} \ g_{(2i)3} \ \dots \ g_{(2i)(2m-1)}\};$$

$$i=2, 4, \dots, (2m),$$

$$G_{22}^T = [g_{221} \ g_{222} \ \dots \ g_{22m}],$$

$$g_{22i} = \{g_{(2i)2} \ g_{(2i)4} \ \dots \ g_{(2i)(2m)}\};$$

$$i=2, 4, \dots, (2m),$$

$$\underline{N}_1 = [n_1 \ n_3 \ \dots \ n_{(2m-1)}]^T,$$

$$\underline{N}_2 = [n_2 \ n_4 \ \dots \ n_{(2m)}]^T.$$

Consider the vector \underline{Z} defined as

$$\underline{Z} = [\underline{V} \ \underline{W}]^T.$$

\underline{Z} can be written as

$$\underline{Z} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \underline{X}_1 \\ \underline{X}_2 \end{bmatrix} + \begin{bmatrix} \underline{N}_1 \\ \underline{N}_2 \end{bmatrix}. \quad (25)$$

Assuming that the signals and noise are statistically independent and that the noise components are uncorrelated from sensor to sensor with covariance matrix $\sigma^2 I_{2m}$ where I_{2m} is the $(2m \times 2m)$ identity matrix. Then $\underline{C}_{zz} = E[\underline{Z} \underline{Z}^H]$ is given by

$$\underline{C}_{zz} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} E[X_1 X_1^H] & E[X_1 X_2^H] \\ E[X_2 X_1^H] & E[X_2 X_2^H] \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^H$$

$$+ \sigma^2 I_{2m} \quad (26)$$

Let $[G]$ be the matrix

$$[G] = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

Then

$$[G]^{-1} (C_{zz} - \sigma^2 I_{2m}) ([G]^{-1})^H =$$

$$\begin{bmatrix} E[X_1 X_1^H] & E[X_1 X_2^H] \\ E[X_2 X_1^H] & E[X_2 X_2^H] \end{bmatrix} \quad (27)$$

Having recovered the matrix on the right side of equation (27), the matrices $M=E[X_1 X_1^H]$ and $N=E[X_1 X_2^H]$ can be identified. Recall that M and N have the decompositions

$$M = ASA^H \text{ and } N = AS\Phi^H A^H, \quad (28)$$

where A , S and Φ are given by

$$\begin{aligned} S &= [S_1^H \ S_2^H] \\ S_1 &= [s_1, \dots, s_d] \text{ impinging signal vector,} \\ A &= [a_1 \ a_2 \ \dots \ a_d] \\ a_i &= [1 \ e^{j2\phi_i} \ \dots \ e^{j(m/2)\phi_i}]^T, \\ \Phi &= \text{diag} \{ e^{j\phi_1}, \dots, e^{j\phi_d} \}, \\ \phi_k &= (\omega d/c) \sin(\theta_k), \quad k=1, 2, \dots, d. \end{aligned}$$

Therefore, the effects of mutual coupling have been eliminated and the rank reducing values of the matrix pencil $(M-\lambda N)$ are given by

$$\lambda_i = e^{j(\omega d/c) \sin(\theta_i)}; \quad i=1, 2, \dots, d. \quad (29)$$

COMPUTER SIMULATION

The scenario used for this simulation consisted of two incoherent sources ($d=2$) which are incident on a linear array consisting of eight half wavelength dipoles ($m=8$). The sources are assumed to be located at $\theta_1=18^\circ$ and $\theta_2=22^\circ$. The noise was simulated to be white Gaussian with zero-mean and unit variance. The sensors were positioned at half wavelength apart such that $\omega d/c = \pi$. 100 snapshots were taken each time and the experiments were repeated 50 times. The results of the simulation are shown below.

First Application

(Without compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	13.1740	70.4337	1.29940	75.60234
25 dB	11.9686	36.3590	11.87542	31.17404
20 dB	14.4098	32.0397	6.606652	12.23856
15 dB	15.4838	31.0915	5.512074	11.46014
10 dB	15.9031	30.8755	7.318329	13.04751

(With compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	18.0652	22.2367	0.330710	0.285269
25 dB	18.0561	22.3543	0.669874	0.748264
20 dB	18.2311	22.5749	1.683848	1.879809
15 dB	18.3796	22.8772	3.496217	4.794093
10dB	18.3964	23.5170	8.045362	9.578419

Second Application

(Without compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	12.6221	29.6850	1.967561	0.634513
25 dB	12.7642	29.7487	9.552104	3.538558
20 dB	14.2713	31.4526	35.51011	51.34790
15 dB	18.4257	40.4587	72.28546	242.0847
10 dB	19.3866	42.4492	66.07262	279.2892

(With compensation for the mutuals)

SNR	mean θ_1	mean θ_2	variance θ_1	variance θ_2
30 dB	18.0593	22.1633	0.071976	0.197370
25 dB	18.0902	22.2343	0.148269	0.514200
20 dB	18.1557	22.4040	0.314386	1.135701
15 dB	18.2589	22.3398	0.737347	2.406417
10 dB	17.9905	22.8160	2.897151	9.306142

Note that extremely poor estimates are obtained without compensation for the mutuals in both cases. Compensation results in significant improvement.

REFERENCES

- [1] A. Paulraj, R. Roy and T. Kailath, "Subspace Rotation Approach To Direction of Arrival Estimation," Nineteenth Annual Asilomar Conf., Pacific Grove, CA., 1981.
- [2] H. Ouibrahim, "A Generalized Approach To Direction Finding," Ph.D. dissertation, Syracuse University, Dec. 1986.
- [3] A. T. Adams, "An Introduction to the Method of Moments," RADC, TP-73-217, Vol. 1, Aug. 1974.
- [4] R. F. Harrington, "Field Computation by Moments Method," Macmillan, N.Y. 1968.